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Inversion of all Principal Submatrices of a Matrix

Let $\mathbf{A}_m$ be an $m \times m$ principal submatrix of an infinite-dimensional matrix $\mathbf{A}$. We give a simple formula which expresses $\mathbf{A}_{m+1}^{-1}$ in terms of $\mathbf{A}_m^{-1}$, and based on this formula, an algorithm which computes the inverses of $\mathbf{A}_m$ for $m = 1,2,3,\ldots,n$ using only $2n^3 - 2n^2 + n$ arithmetic operations. This is an improvement over the naive method of computing the inverses separately which would require $\sum_{m=1}^{n} m^3 = O(n^4)$ arithmetic operations.

I. THE MOTIVATION

The following problem is frequently encountered in many aerospace engineering calculations, such as the Kalman filtering algorithm, as well as in signal processing and systems control theory [2, 3, 1, 4]. Given a tolerance $\epsilon > 0$ and an infinite-dimensional linear system of equations of the form

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots \\ a_{21} & a_{22} & a_{23} & \cdots \\ a_{31} & a_{32} & a_{33} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \end{bmatrix}$$

find a finite-dimensional solution vector $\mathbf{x}_n = [x_1, x_2, \ldots, x_n]^T$ such that

$$f(\mathbf{x}_n) \le \epsilon$$

for some nonnegative objective functional f. This can be achieved by solving the following sequence of (increasing) finite-dimensional problems:

$$[a_{11}][x_1] = [b_1]$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\vdots$$

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

and checking if the above tolerance criterion is satisfied. Solving this sequence of finite-dimensional linear systems is equivalent to computing the following sequence of matrix inversions:

$$\mathbf{A}_1^{-1}, \mathbf{A}_2^{-1}, \mathbf{A}_3^{-1}, \ldots, \mathbf{A}_n^{-1}$$

where $\mathbf{A}_m$ for $1 \le m \le n$ is an $m \times m$ principal submatrix of the infinite-dimensional system matrix $\mathbf{A} = [a_{ij}]_{i,j=1}^{\infty}$.

II. THE FORMULA

We partition the $(m+1) \times (m+1)$ matrix $\mathbf{A}_{m+1}$ as follows

$$\mathbf{A}_{m+1} = \left[\begin{array}{ccc|c} a_{11} & \cdots & a_{1m} & a_{1,m+1} \\ \vdots & \ddots & \vdots & \vdots \\ a_{m1} & \cdots & a_{mm} & a_{m,m+1} \\ \hline a_{m+1,1} & \cdots & a_{m+1,m} & a_{m+1,m+1} \end{array}\right] := \begin{bmatrix} \mathbf{A}_m & \mathbf{b} \\ \mathbf{c}^T & a \end{bmatrix}.$$

We assume that $\mathbf{A}_m^{-1}$ exists and

$$d := a - \mathbf{c}^T \mathbf{A}_m^{-1} \mathbf{b} \neq 0.$$

Let $\alpha := 1/d$. We claim that the inverse of $\mathbf{A}_{m+1}$ is given by

$$\mathbf{A}_{m+1}^{-1} = \begin{bmatrix} \mathbf{A}_m^{-1} + \alpha \mathbf{A}_m^{-1} \mathbf{b} \mathbf{c}^T \mathbf{A}_m^{-1} & -\alpha \mathbf{A}_m^{-1} \mathbf{b} \\ -\alpha \mathbf{c}^T \mathbf{A}_m^{-1} & \alpha \end{bmatrix}.$$

This claim is easily verified as follows:

$$\begin{aligned} &\mathbf{A}_{m+1}\mathbf{A}_{m+1}^{-1} \\ &= \begin{bmatrix} \mathbf{A}_m & \mathbf{b} \\ \mathbf{c}^T & a \end{bmatrix} \begin{bmatrix} \mathbf{A}_m^{-1} + \alpha \mathbf{A}_m^{-1} \mathbf{b} \mathbf{c}^T \mathbf{A}_m^{-1} & -\alpha \mathbf{A}_m^{-1} \mathbf{b} \\ -\alpha \mathbf{c}^T \mathbf{A}_m^{-1} & \alpha \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{I}_m + \alpha \mathbf{b} \mathbf{c}^T \mathbf{A}_m^{-1} - \alpha \mathbf{b} \mathbf{c}^T \mathbf{A}_m^{-1} & -\alpha \mathbf{b} + \alpha \mathbf{b} \\ \mathbf{c}^T \mathbf{A}_m^{-1} + \alpha \mathbf{c}^T \mathbf{A}_m^{-1} \mathbf{b} \mathbf{c}^T \mathbf{A}_m^{-1} - \alpha a \mathbf{c}^T \mathbf{A}_m^{-1} & -\alpha \mathbf{c}^T \mathbf{A}_m^{-1} \mathbf{b} + \alpha a \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{I}_m & \mathbf{0} \\ \mathbf{0}^T & 1 \end{bmatrix} = \mathbf{I}_{m+1}. \end{aligned}$$

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TABLE I
Computation of $\mathbf{A}_{m+1}^{-1}$ Given $\mathbf{A}_m^{-1}$

Step	Operation	Complexity
1.	$(\mathbf{A}_m^{-1})(\mathbf{b})$	$m(2m-1)$
2.	$(\mathbf{c}^T)(\mathbf{A}_m^{-1})$	$m(2m-1)$
3.	$(\mathbf{c}^T\mathbf{A}_m^{-1})(\mathbf{b})$	$2m-1$
4.	$(a)-(\mathbf{c}^T\mathbf{A}_m^{-1}\mathbf{b})$	1
5.	$\alpha := 1/(a-\mathbf{c}^T\mathbf{A}_m^{-1}\mathbf{b})$	1
6.	$(-\alpha)(\mathbf{A}_m^{-1}\mathbf{b})$	m
7.	$(-\alpha)(\mathbf{c}^T\mathbf{A}_m^{-1})$	m
8.	$(\alpha\mathbf{A}_m^{-1}\mathbf{b})(\mathbf{c}^T\mathbf{A}_m^{-1})$	m^2
9.	$(\mathbf{A}_m^{-1})+(\alpha\mathbf{A}_m^{-1}\mathbf{b}\mathbf{c}^T\mathbf{A}_m^{-1})$	m^2

It is also straightforward to show that $\mathbf{A}_{m+1}^{-1}\mathbf{A}_{m+1} = \mathbf{I}_{m+1}$.

III. THE ALGORITHM

The algorithm computes the inverses in the following order: $\mathbf{A}_1^{-1}, \mathbf{A}_2^{-1}, \mathbf{A}_3^{-1}, \ldots, \mathbf{A}_n^{-1}$. Once $\mathbf{A}_m^{-1}$ is obtained, we compute $\mathbf{A}_{m+1}^{-1}$ using some additional vector-vector, vector-matrix, and matrix-matrix operations. We give the steps of the algorithm in Table I, where the number of arithmetic operations required at each step is also indicated.

Let $T(m)$ be the number of arithmetic operations required to compute the inverse of $\mathbf{A}_m$. Assuming that $\mathbf{A}_m^{-1}$ is already computed using $T(m)$ arithmetic operations, we proceed to compute $\mathbf{A}_{m+1}^{-1}$ using additional $6m^2+2m+1$ arithmetic operations, as can be seen from Table I. Note that $\mathbf{A}_1$ is only a scalar, thus $T(1)=1$. As m runs from 2 to n, we obtain the inverses of all $\mathbf{A}_m$ for $2 \le m \le n$ using

$$T(n) = T(n-1) + 6(n-1)^2 + 2(n-1) + 1$$

arithmetic operations. Thus, the number of arithmetic operations required for computing $\mathbf{A}_m^{-1}$ for $1 \le m \le n$ is found as

$$\begin{aligned} T(n) &= T(n-1) + 6(n-1)^2 + 2(n-1) + 1 \\ &= T(n-2) + 6(n-2)^2 + 6(n-1)^2 \\ &\quad + 2(n-2) + 2(n-1) + 1 + 1 \\ &\vdots \\ &= T(1) + 6\sum_{m=1}^{n-1} m^2 + 2\sum_{m=1}^{n-1} m + \sum_{m=1}^{n-1} 1 \\ &= 1 + (n-1)n(2n-1) + (n-1)n + (n-1) \\ &= 2n^3 - 2n^2 + n. \end{aligned}$$

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Application of Neural Networks in Target Tracking Data Fusion

Kalman filtering is a fundamental building block of most multiple-target tracking (MTT) algorithms. The other building block usually involves some type of data association schemes. Here it is proposed to incorporate a neural network into the normal Kalman filter configuration such that the neural network provides the adaptive capability the filter needs. As such, the estimation error of the Kalman filter would be reduced, hence improving the MTT solution. Simulation results have shown that this claim is valid.

I. INTRODUCTION

Target tracking is an important issue in military surveillance systems, especially when such systems employ multiple sensors to interpret the environment. Typical sensors such as radar, infrared, electronic support measure, sonar, etc. report measurements from various sources including target kinematics, attributes, clutter and background noise. The objective of target tracking, or in general, multiple-target tracking (MTT) is to partition sensor data into sets of observations, or tracks produced by the same source. Once tracks are formed and confirmed, the number of targets can be estimated and parameters such as position and velocity can be obtained from each track. When multiple sensors are used, the

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